Key

3.18 DeMoivre's Theorem & nth Roots

Name____________________________ Date __________ Period ___________

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Express the complex number in trigonometric form. $r(\cos\theta + i\sin\theta)$, $r=\sqrt{a^2+b^2}$, $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

1) 3i point on y-axis — 1) ________

$a=3$, $b=0$

$r=3$, $\theta = \tan^{-1}(0) = \frac{\pi}{2}$

$\boxed{3\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)}$

2) 2 + 2i → point in quad. 1 — 2) ________

$a=2$, $b=2$

$r=\sqrt{8}=2\sqrt{2}$, $\theta=\tan^{-1}(1)=\pi/4$

$\boxed{2\sqrt{2}\left(\cos\pi/4 + i\sin\pi/4\right)}$

3) $-2 + 2\sqrt{3}i$ → point in quad. 2 — 3) ________

$a=-2$, $b=2\sqrt{3}$, $r=4$

$\theta=\tan^{-1}(-\sqrt{3}) = \frac{2\pi}{3}$ (in quad. 2)

$\boxed{4\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)}$

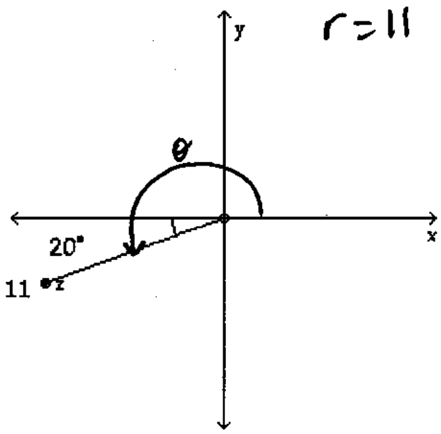
4) 3 + 2i → quad. 1 — 4) ________

$a=3$, $b=2$, $r=\sqrt{13}$

$\theta=\tan^{-1}\left(\frac{2}{3}\right) \approx 33.7^\circ$

$\boxed{\sqrt{13}\left(\cos 33.7^\circ + i\sin 33.7^\circ\right)}$

5) — 5) ________



$r=11$, $\theta = 180^\circ + 20^\circ = 200^\circ$

$\boxed{11\left(\cos 200^\circ + i\sin 200^\circ\right)}$

Write the complex number in the form a + bi.

$a = r\cos\theta$

$b = r\sin\theta$

6) 3(cos 30° + i sin 30°) — 6) ________

$a = 3\cos 30^\circ = 3\cdot\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}$

$b = 3\sin 30^\circ = 3\cdot\frac{1}{2} = \frac{3}{2}$

$\boxed{\frac{3\sqrt{3}}{2} + \frac{3}{2}i}$

7) $5(\cos -60° + i \sin -60°)$ 7) ____________

$a = 5\cos(-60°) = 5 \cdot \frac{1}{2} = \frac{5}{2}$

$b = 5\sin(-60°) = 5 \cdot -\frac{\sqrt{3}}{2} = \frac{-5\sqrt{3}}{2}$

$\boxed{\frac{5}{2} - \frac{5\sqrt{3}}{2} i}$

8) $\sqrt{2}\left(\cos\frac{\pi}{6} + i \sin\frac{\pi}{6}\right)$ 8) ____________

$a = \sqrt{2}\cos\frac{\pi}{6} = \sqrt{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{6}}{2}$

$b = \sqrt{2}\sin\frac{\pi}{6} = \sqrt{2} \cdot \frac{1}{2} = \frac{\sqrt{2}}{2}$

$\boxed{\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} i}$

Find the product or quotient, as indicated. Leave your answer in trigonometric form.

9) Find the product of z_1 and z_2. $z_1 \cdot z_2 = r_1 \cdot r_2 (\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2))$ ____________

$z_1 = 7(\cos 25° + i \sin 25°)$, $z_2 = 2(\cos 130° + i \sin 130°)$

$z_1 \cdot z_2 = 7 \cdot 2(\cos(25° + 130°) + i \sin(25° + 130°))$

$= \boxed{14(\cos 155° + i \sin 155°)}$

10) Find the product of z_1 and z_2. 10) ____________

$z_1 = 5\left(\cos\frac{\pi}{4} + i \sin\frac{\pi}{4}\right)$, $z_2 = 3\left(\cos\frac{5\pi}{3} + i \sin\frac{5\pi}{3}\right)$

$z_1 \cdot z_2 = \boxed{15\left(\cos\frac{23\pi}{12} + i \sin\frac{23\pi}{12}\right)}$

11) Find the quotient. $\frac{z_1}{z_2} = \frac{r_1}{r_2}(\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2))$ 11) ____________

$\frac{2(\cos 30° + i \sin 30°)}{3(\cos 60° + i \sin 60°)}$

$\frac{z_1}{z_2} = \frac{2}{3}(\cos(30° - 60°) + i \sin(30° - 60°))$

$= \frac{2}{3}\cos(-30°) + i \sin(-30°))$

$= \boxed{\frac{2}{3}(\cos(30°) - i \sin(30°))}$

12) Find the quotient. 12) ____________

$$\frac{6(\cos 5\pi + i \sin 5\pi)}{3(\cos 2\pi + i \sin 2\pi)}$$

$\frac{z_1}{z_2} = 2(\cos 3\pi + i\sin 3\pi) = \boxed{2(\cos\pi + i\sin\pi)}$

Find the product and quotient in two ways, a) using the trigonometric form for z1 and z2 , and b) using the standard form for z1 and z2.

13) z1 = 3 − 2i z2 = 1 + i 13) ____________

a) $z_1 = \sqrt{13}(\cos 326° + i\sin 326°)$

$z_2 = \sqrt{2}(\cos 45° + i\sin 45°)$

$\boxed{z_1 \cdot z_2 = \sqrt{26}(\cos 371° + i\sin 371°)}$

$\boxed{\frac{z_1}{z_2} = \frac{\sqrt{13}}{\sqrt{2}} = (\cos 281° + i\sin 281°)}$

b) $(3-2i)(1+i) =$

$= 3 + 3i - 2i - 2i^2$

$= 3 + i + 2 = \boxed{5+i}$

$\frac{z_1}{z_2} = \frac{3-2i}{1+i}\left(\frac{1-i}{1-i}\right) = \frac{3-3i-2i+2i^2}{2} = \frac{1-5i}{2} = \boxed{\frac{1}{2} - \frac{5}{2}i}$

14) z1 = 3 + i, z2 = 5 − 3i 14) ____________

$z_1 = \sqrt{10}(\cos 18.4° + i\sin 18.4°)$

$z_2 = \sqrt{34}(\cos 329° + i\sin 329°)$

$\boxed{z_1 \cdot z_2 = \sqrt{340}(\cos 347.4° + i\sin 347.4°)}$

$\boxed{\frac{z_1}{z_2} = \frac{\sqrt{10}}{\sqrt{34}}(\cos 49.4° + i\sin 49.4°)}$

b) $(3+i)(5-3i) = \boxed{18-4i}$

$\frac{3+i}{5-3i}\left(\frac{5+3i}{5+3i}\right) = \boxed{\frac{6}{17} + \frac{7}{17}i}$

Use De Moivre's Theorem to find the indicated power of the complex number. Write your answer in standard form a + bi.

15) $(\cos \pi/4 + i \sin \pi/4)^3$ 15) ____________

$z^n = r^n(\cos(n\theta) + i\sin(n\theta))$

$\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right) = \boxed{-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i}$

16) $(2(\cos 3\pi/4 + i \sin 3\pi/4))^3$ 16) ____________

$2^3\left(\cos\left(3\cdot\frac{3\pi}{4}\right) + i\sin\left(3\cdot\frac{3\pi}{4}\right)\right)$

$= 8\left(\cos\frac{9\pi}{4} + i\sin\frac{9\pi}{4}\right) = 8\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$= 8\cdot\frac{\sqrt{2}}{2} + i\,8\frac{\sqrt{2}}{2} = \boxed{4\sqrt{2} + 4\sqrt{2}i}$

17) $(1 + i)^5$ 17) ____________

Change to trig-form first!

$z = 1+i = \sqrt{2}(\cos\pi/4 + i\sin\frac{\pi}{4})$

$z^5 = (\sqrt{2})^5\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$

$= \sqrt{2}^5 \cdot -\frac{\sqrt{2}}{2} + \sqrt{2}^5 \cdot -\frac{\sqrt{2}}{2} = -\frac{\sqrt{2}^6}{2} + -\frac{\sqrt{2}^6}{2}i$

$= \frac{-8}{2} + \frac{-8}{2}i = \boxed{-4+4i}$

write in trig form.

18) $(1-\sqrt{3}i)^3$ $z = 2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$ 18) ______

$z^3 = 2^3(\cos 5\pi + i\sin 5\pi)$

$= 8(\cos\pi + i\sin\pi) = 8\cdot(-1) + 8(0)i = \boxed{-8}$

Find the indicated roots. Write the answer in trigonometric form.

19) Cube roots of $2(\cos 2\pi + i\sin 2\pi)$ First angle $2\pi\cdot\frac{1}{3} = \frac{2\pi}{3}$ add $\frac{2\pi}{3}$ to each angle. 19) ______

$z_1 = \sqrt[3]{2}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

$z_2 = \sqrt[3]{2}\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$

$z_3 = \sqrt[3]{2}(\cos 2\pi + i\sin 2\pi)$

20) Cube roots of $3\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$ First angle $\frac{4\pi}{3}\cdot\frac{1}{3} = \frac{4\pi}{9}$ add $\frac{2\pi}{3}$ to each angle. 20) ______

$z_1 = \sqrt[3]{3}\left(\cos\frac{4\pi}{9} + i\sin\frac{4\pi}{9}\right)$

$z_2 = \sqrt[3]{3}\left(\cos\frac{10\pi}{9} + i\sin\frac{10\pi}{9}\right)$

$z_3 = \sqrt[3]{3}\left(\cos\frac{16\pi}{9} + i\sin\frac{16\pi}{9}\right)$

Find the indicated roots. Write the answer in a + bi form.

21) Cube roots of $3 - 4i$ $r = 5$ $\theta = 306.9°$ First angle $306.9°\cdot\frac{1}{3} = 102.3°$ add $120°$ 21) ______

$z = 5(\cos 306.9° + i\sin 306.9°)$

$z_1 = \sqrt[3]{5}(\cos 102.3° + i\sin 102.3°) = \boxed{-.36 + 1.67i}$

$z_2 = \sqrt[3]{5}(\cos 222.3° + i\sin 222.3°) = \boxed{-1.26 - 1.15i}$

$z_3 = \sqrt[3]{5}(\cos 342.3° + i\sin 342.3°) = \boxed{1.63 - .52i}$

Find the indicated roots. Write the answer in trigonometric form.

22) Fifth roots of $(\cos\pi + i\sin\pi)$ First angle $\frac{\pi}{5}$ add $\frac{2\pi}{5}$ to each angle. 22) ______

$z_1 = \cos\frac{\pi}{5} + i\sin\frac{\pi}{5}$

$z_2 = \cos\frac{3\pi}{5} + i\sin\frac{3\pi}{5}$

$z_3 = \cos\pi + i\sin\pi$

$z_4 = \cos\frac{7\pi}{5} + i\sin\frac{7\pi}{5}$

$z_5 = \cos\frac{9\pi}{5} + i\sin\frac{9\pi}{5}$

1^{st} angle $\frac{\pi}{6}\cdot\frac{1}{5}=\frac{\pi}{30}$ add $\frac{2\pi}{5}$ to each angle.

23) Fifth roots of $2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$ 23) ____________

$z_1=\sqrt[5]{2}\left(\cos\frac{\pi}{30}+i\sin\frac{\pi}{30}\right)$

$z_2=\sqrt[5]{2}\left(\cos\frac{13\pi}{30}+i\sin\frac{13\pi}{30}\right)$

$z_3=\sqrt[5]{2}\left(\cos\frac{25\pi}{30}+i\sin\frac{25\pi}{30}\right)$

$z_4=\sqrt[5]{2}\left(\cos\frac{37\pi}{30}+i\sin\frac{37\pi}{30}\right)$

$z_5=\sqrt[5]{2}\left(\cos\frac{49\pi}{30}+i\sin\frac{49\pi}{30}\right)$

Find the indicated roots. Write the answer in a + bi form.

24) Fifth roots of 2i $r=2$ $\theta=\frac{\pi}{2}$ $z=2(\cos\pi/2+i\sin\pi/2)$ 24) ____________

$z_1=\sqrt[5]{2}\left(\cos\frac{\pi}{10}+i\sin\frac{\pi}{10}\right)=\boxed{1.09+.35i}$

$z_2=\sqrt[5]{2}\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)=\boxed{0+1.15i}$

$z_3=\sqrt[5]{2}\left(\cos\frac{9\pi}{10}+i\sin\frac{9\pi}{10}\right)=\boxed{-1.09+.35i}$

$z_4=\sqrt[5]{2}\left(\cos\frac{13\pi}{10}+i\sin\frac{13\pi}{10}\right)=\boxed{-.68-.93i}$

$z_5=\sqrt[5]{2}\left(\cos\frac{17\pi}{10}+i\sin\frac{17\pi}{10}\right)=\boxed{.68-.93i}$

Express the indicated roots of unity in standard form a + bi.

25) Cube roots of unity $z=1$ 25) ____________

$z_1=\cos 0+i\sin 0=\boxed{1+0i}$

$z_2=\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}=\boxed{-\frac{1}{2}+\frac{\sqrt{3}}{2}i}$

$z_3=\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}=\boxed{-\frac{1}{2}-\frac{\sqrt{3}}{2}i}$

26) Sixth roots of unity $z=1$ 26) ____________

$z_1=\cos 0+i\sin 0=\boxed{1+0i}$

$z_2=\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}=\boxed{\frac{1}{2}+\frac{\sqrt{3}}{2}i}$

$z_3=\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}=\boxed{-\frac{1}{2}+\frac{\sqrt{3}}{2}i}$

$z_4=\cos\pi+i\sin\pi=\boxed{-1+0i}$

$z_5=\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}=\boxed{-\frac{1}{2}-\frac{\sqrt{3}}{2}i}$

$z_6=\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3}=\boxed{\frac{1}{2}-\frac{\sqrt{3}}{2}i}$